Statistical Mechanics

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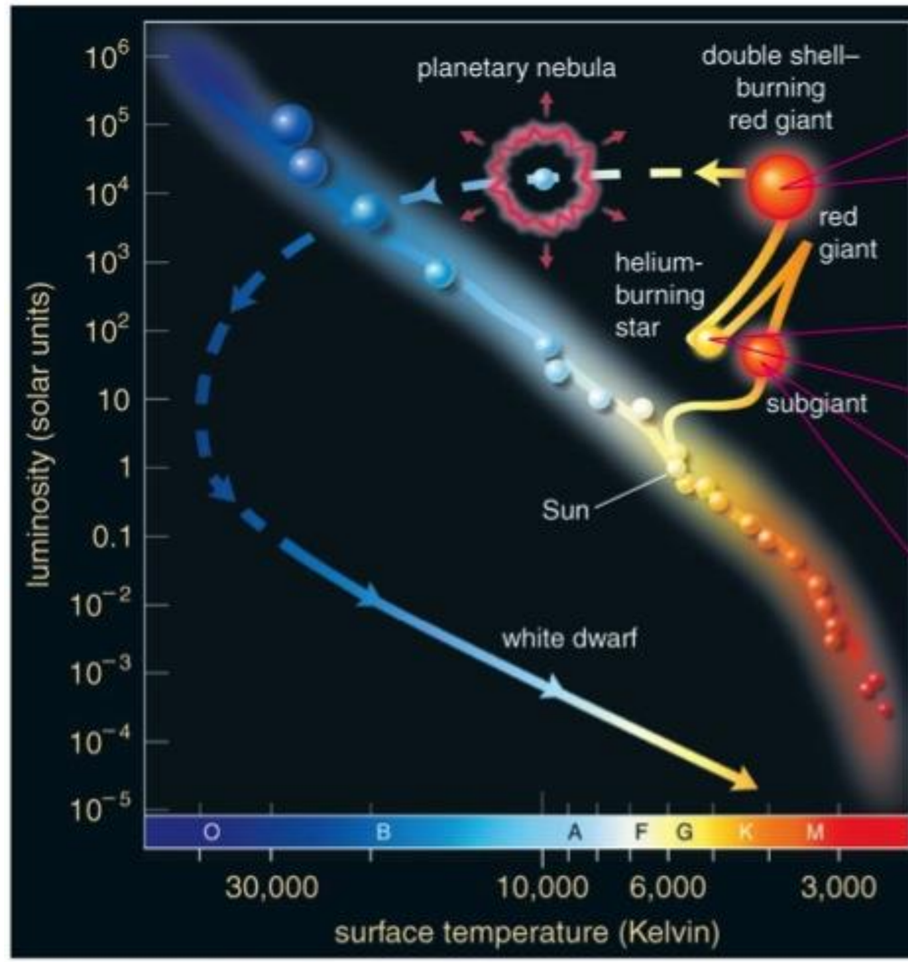
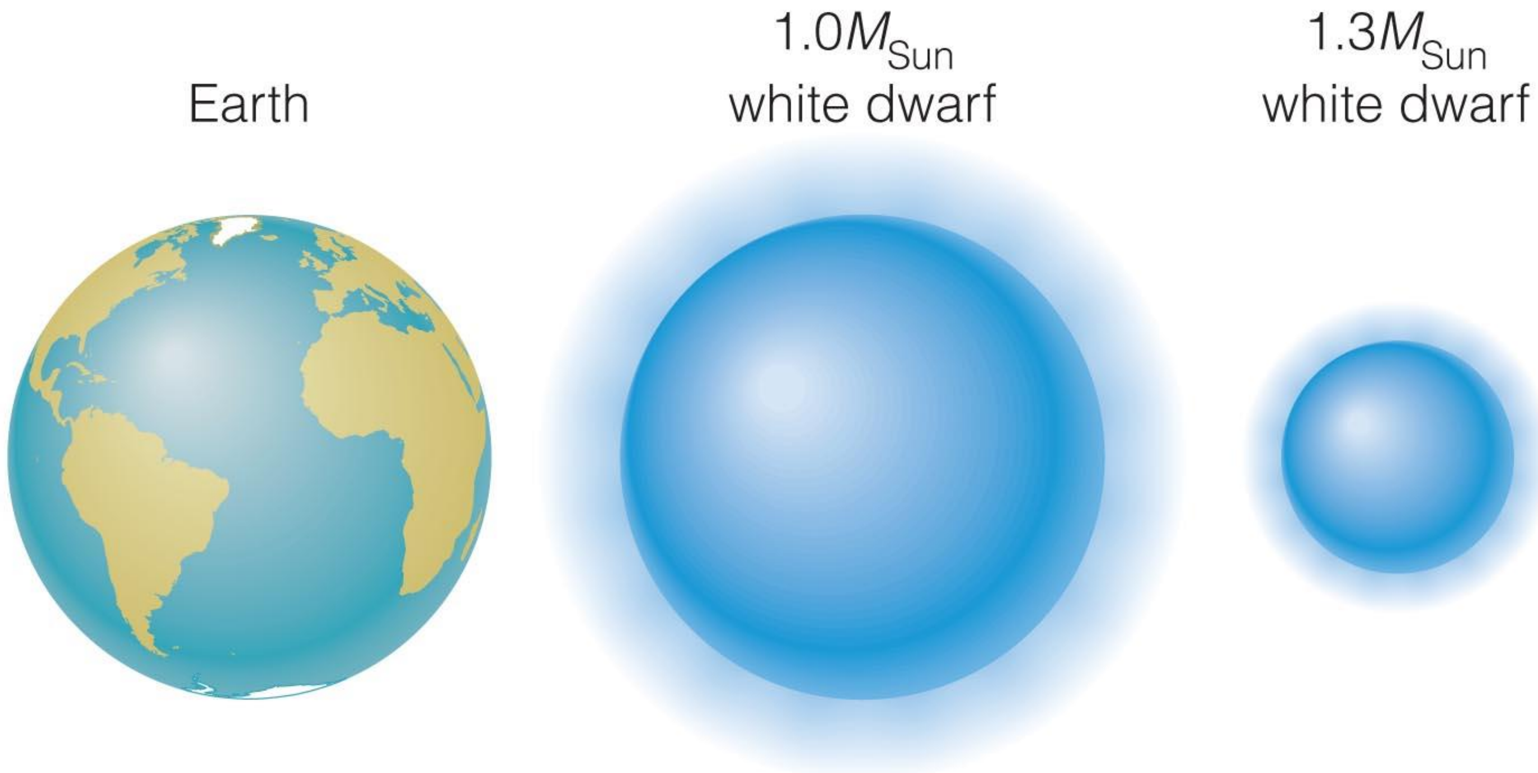
White Dwarfs

- White dwarfs are the remaining cores of dead stars.
- Electron degeneracy pressure supports them against the crush of gravity.
- White dwarfs cool off and grow dimmer with time.



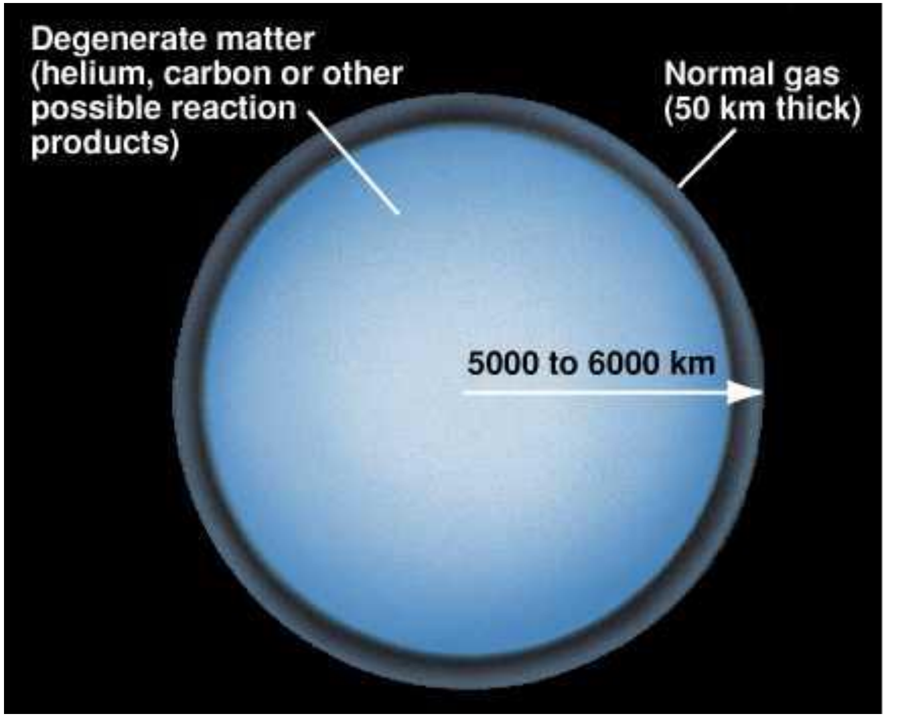
Size of a White Dwarf



- White dwarfs with same mass as Sun are about same size as Earth.
- Higher-mass white dwarfs are smaller.

Composition

- Heavy nuclei are pulled below the surface, while hydrogen rises to the top, layered above the helium



The White Dwarf Limit

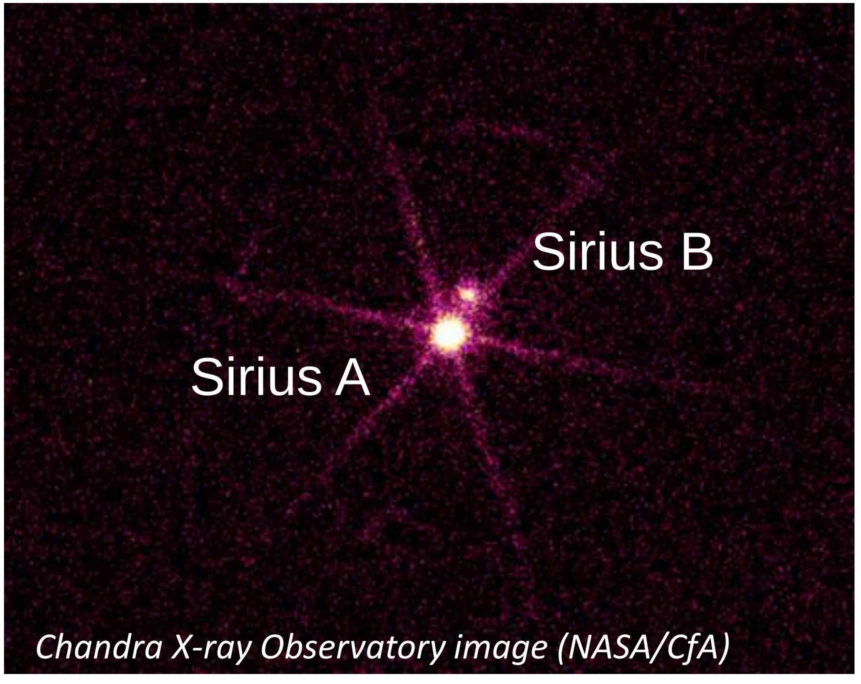
- Quantum mechanics says that electrons must move faster as they are squeezed into a very small space.
- As a white dwarf's mass approaches $1.4M_{\text{Sun}}$, its electrons must move at nearly the speed of light.
- Because nothing can move faster than light, a white dwarf cannot be more massive than $1.4M_{\text{Sun}}$, the *white dwarf limit* (or *Chandrasekhar limit*).

Properties of White Dwarf Stars

- Mass of white dwarf = Mass of Sun = 10^{33} gm
- Content : Helium
- Temperature = 10^{7} K
- Density= 10^{7} gm/cc = 10 times that of sun
- Much Fainter than sun

White Dwarf Stars

- Discovered in 1862,
- They were a hot topic in astronomy in the 1920s.
- Thousands are known today.



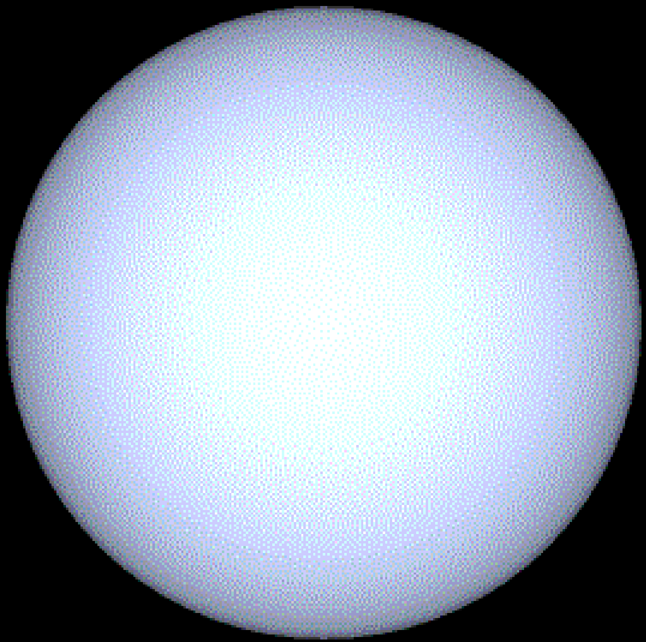
Chandra X-ray Observatory image (NASA/CfA)

• Sirius, the brightest star in the sky, has a companion star which is a white dwarf.

• They orbit each other with a period of about 50 years.

• Sirius A is vastly brighter than Sirius B at visible wavelengths; the contrast is smaller in this X-ray image.

White Dwarf



Sirius B

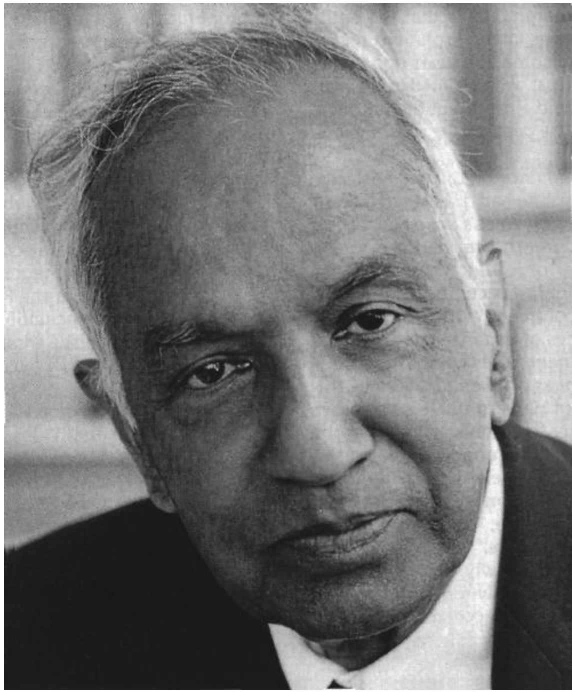
$M \approx 1.0\ M_{sun}$

$R \approx 5800$ km

Chandrasekhar’s theory of white dwarfs

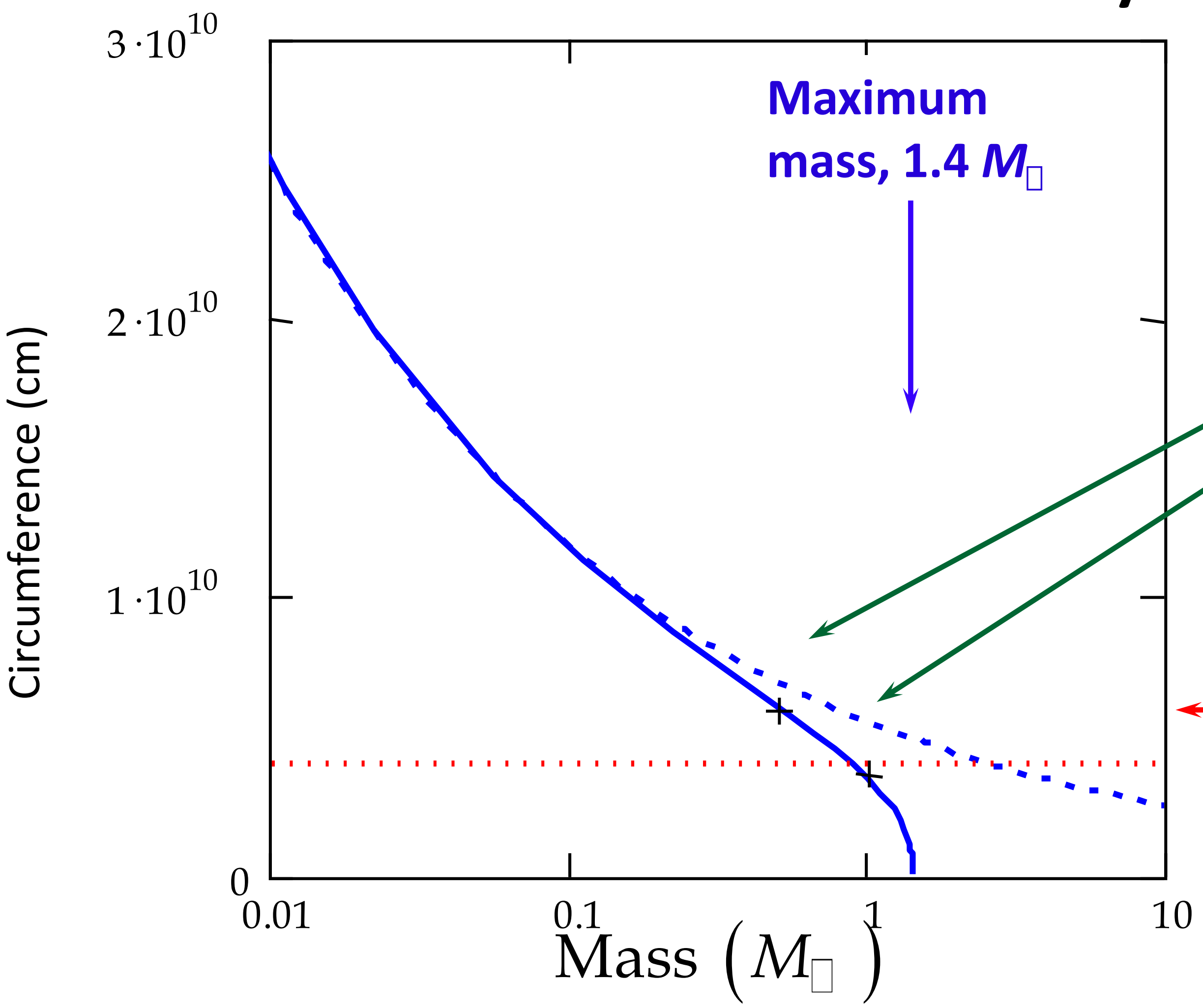
- For stars heavier than about a solar mass, Chandrasekhar found from his theory that the confinement imparted so much energy to the electrons in the center of the star that the electron speeds are close to the speed of light.

- Fowler’s theory of degenerate matter did not take Einstein’s special theory of relativity into account; therefore Chandrasekhar had to start over, and combine relativity and quantum mechanics into a new theory of **relativistic degeneracy pressure**.



- Recall that electrons, like everything else, can't move faster than light.
  - The more massive the degenerate star, the closer the electron speeds get to the speed of light.
  - Closer the speeds get to *c*, the harder it is to accelerate the electrons further.
  - Thus, the electron degeneracy pressure doesn't keep increasing as much with tighter confinement: the electrons reach a point where they cannot move any faster. There is a maximum to the electron degeneracy pressure, and a corresponding maximum weight that degeneracy pressure can support.
  - If the weight cannot be supported by electron degeneracy pressure, the degenerate star will collapse to smaller sizes.

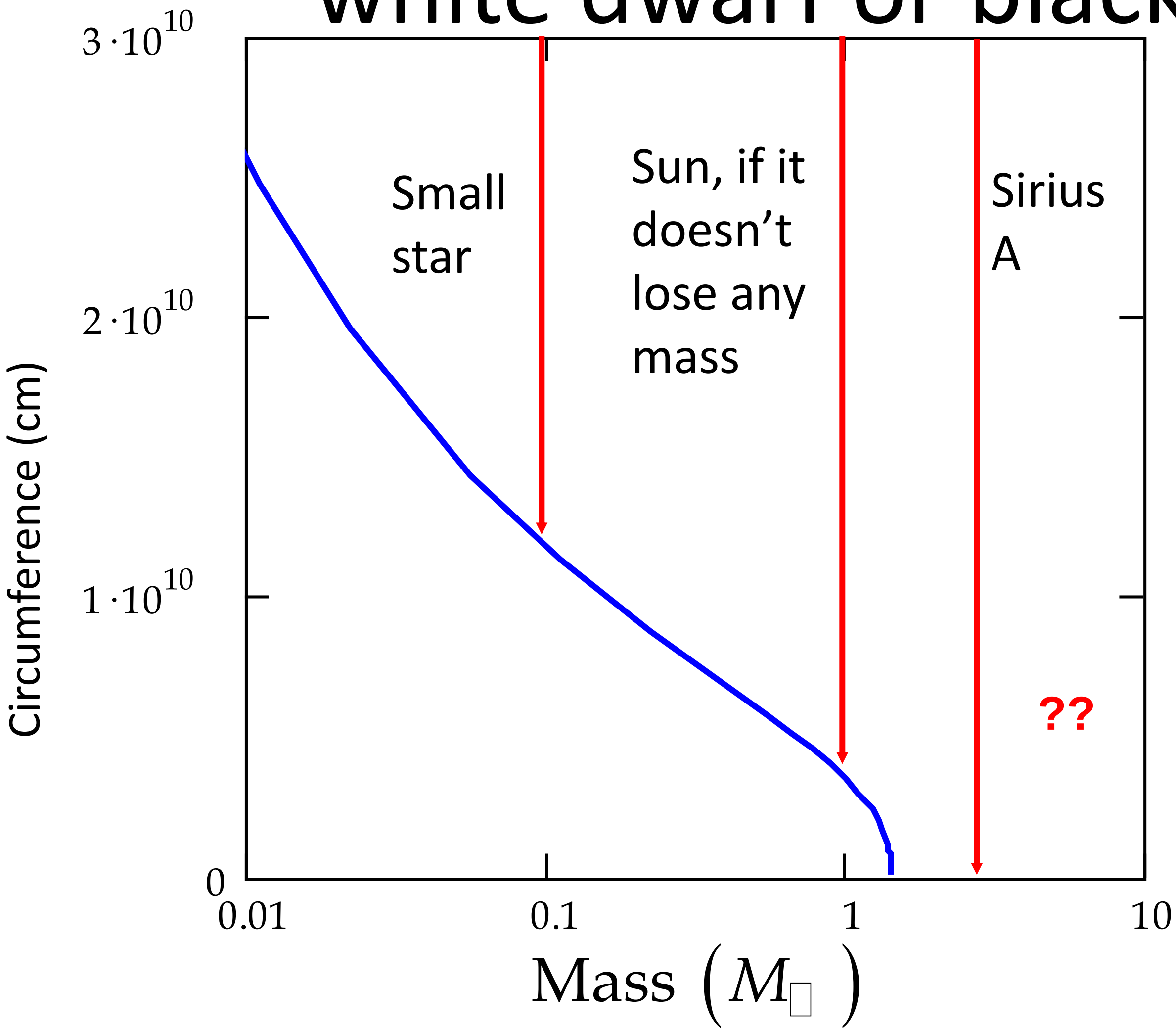
Chandrasekhar's relativistic white dwarf theory



1997 measurements of white dwarf mass and circumference by Provencal *et al.:*
40 Eridani B
Sirius B

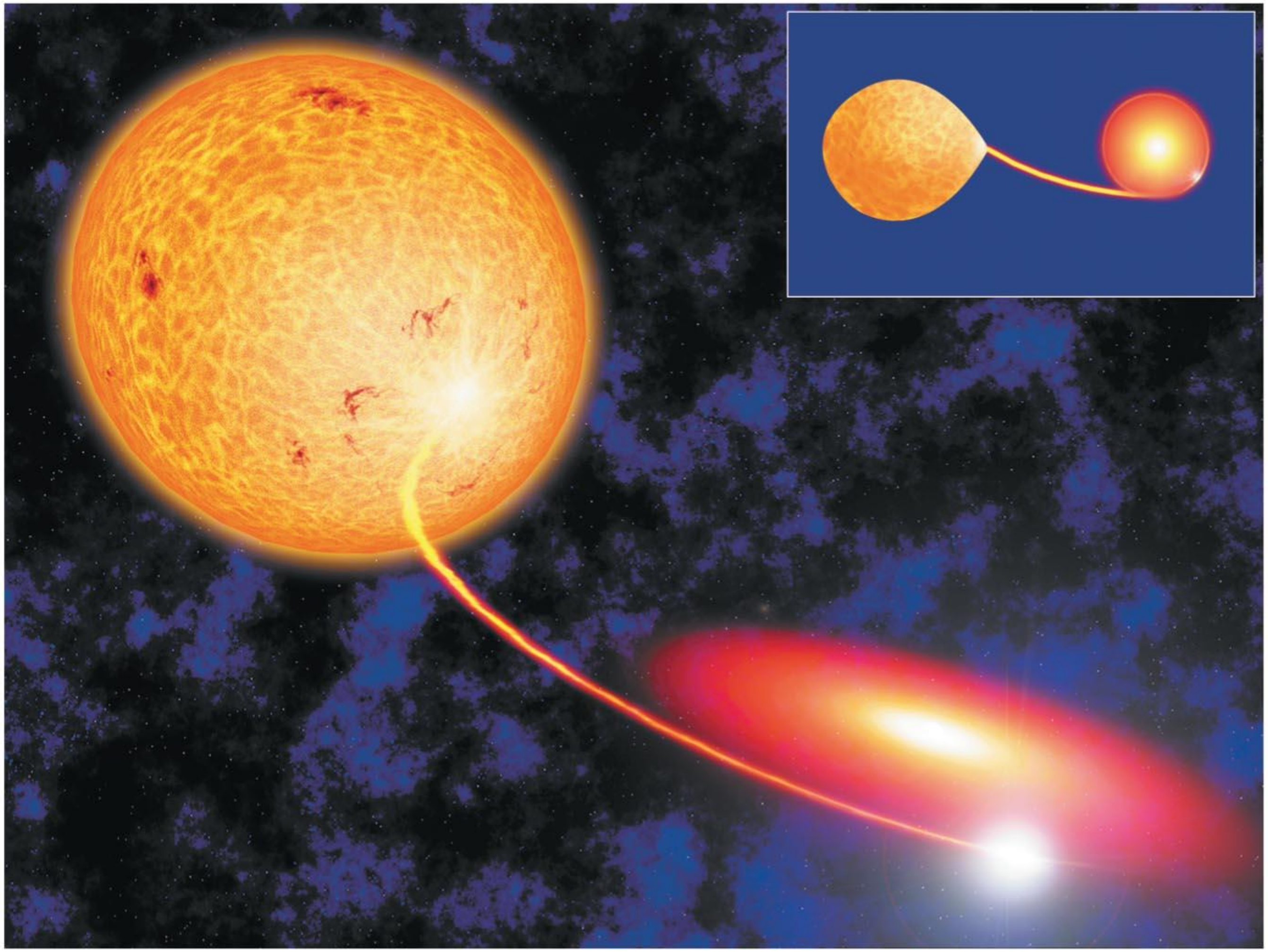
Earth's circumference

Final collapse of burned-out stars: white dwarf or black hole?

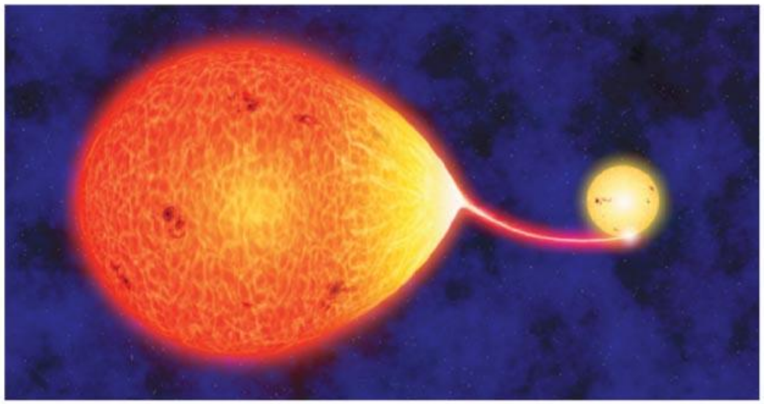


The Sun and smaller stars will become white dwarfs after they burn out and, lacking the gas pressure generated by their nuclear heat source, collapse under their weight. What about Sirius A, which weighs a good deal more than the limit?

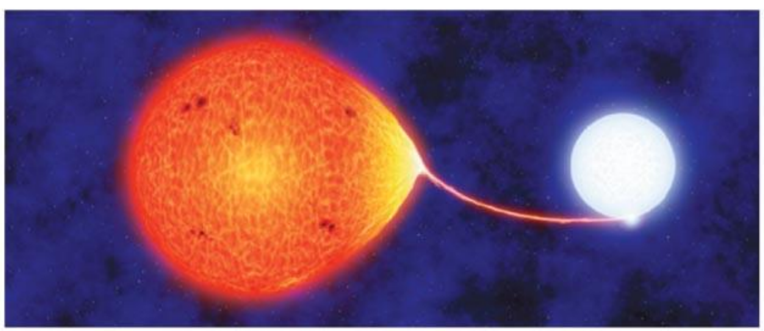
What can happen to a white dwarf in a close binary system?



Algol shortly after its birth. The higher-mass star (left) evolved more quickly than its lower-mass companion (right).



Algol at onset of mass transfer. When the more massive star expanded into a red giant, it began losing some of its mass to its normal, hydrogen-burning companion.



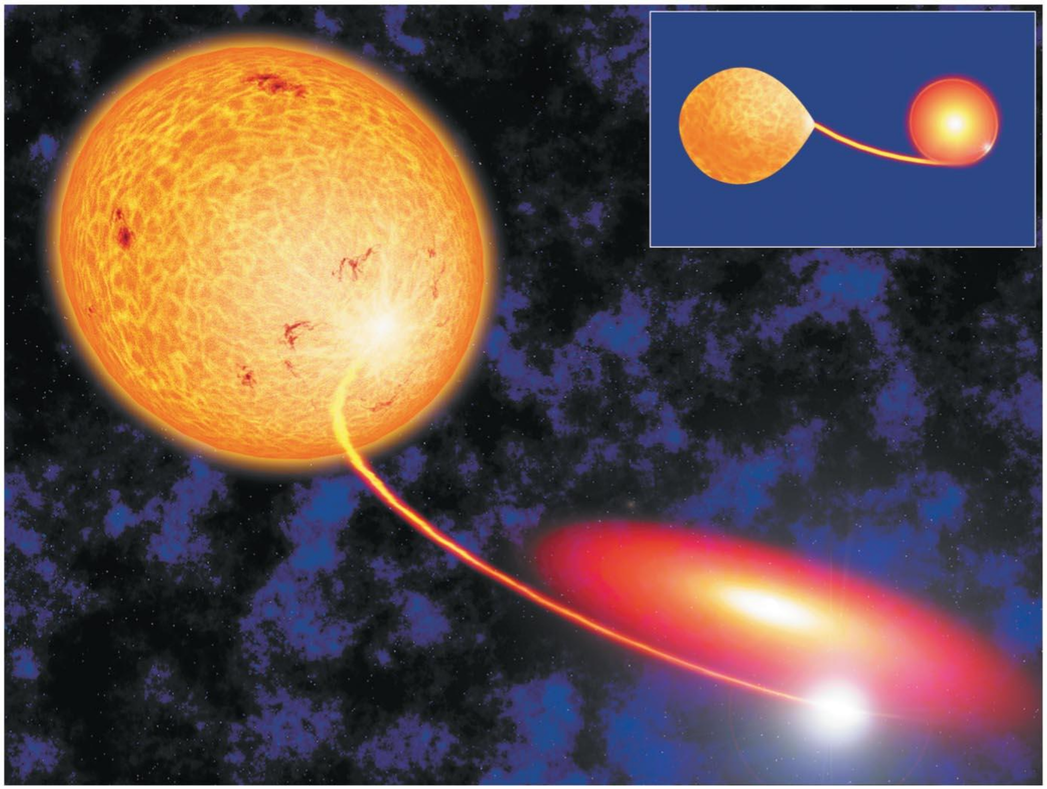
Algol today. As a result of the mass transfer, the red giant has shrunk to a subgiant, and the normal star on the right is now the more massive of the two stars.

A star that started with less mass gains mass from its companion.

Eventually, the mass-losing star will become a white dwarf.

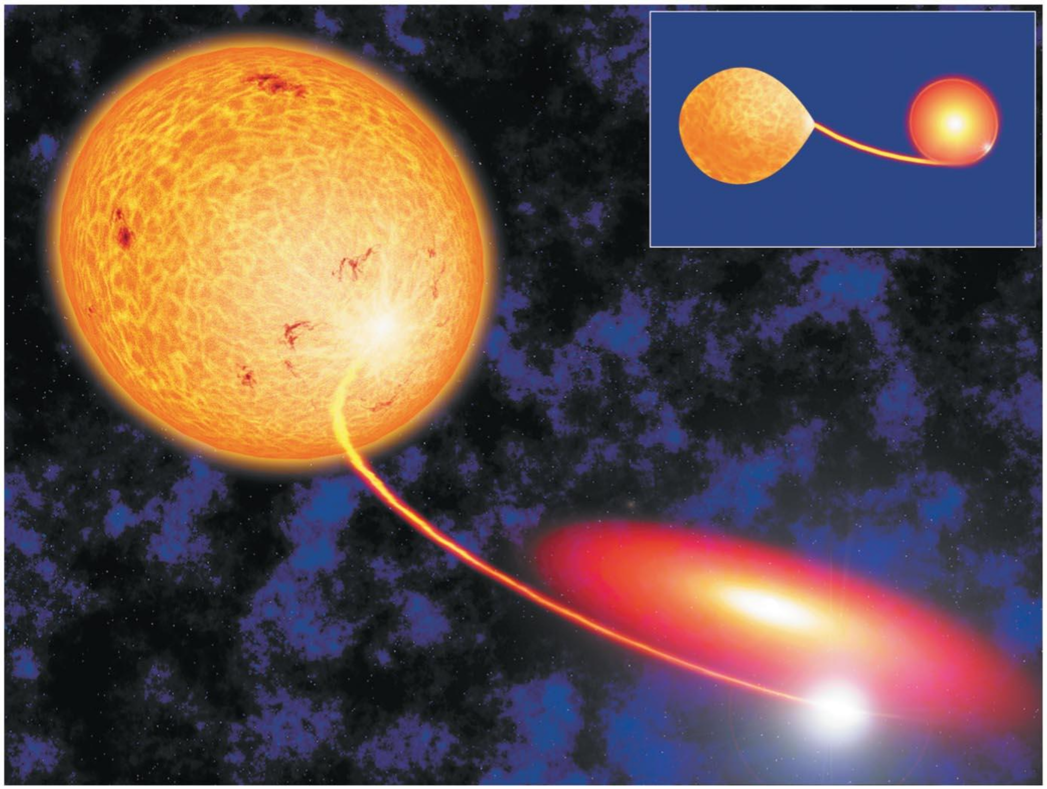
What happens next?

Accretion Disks



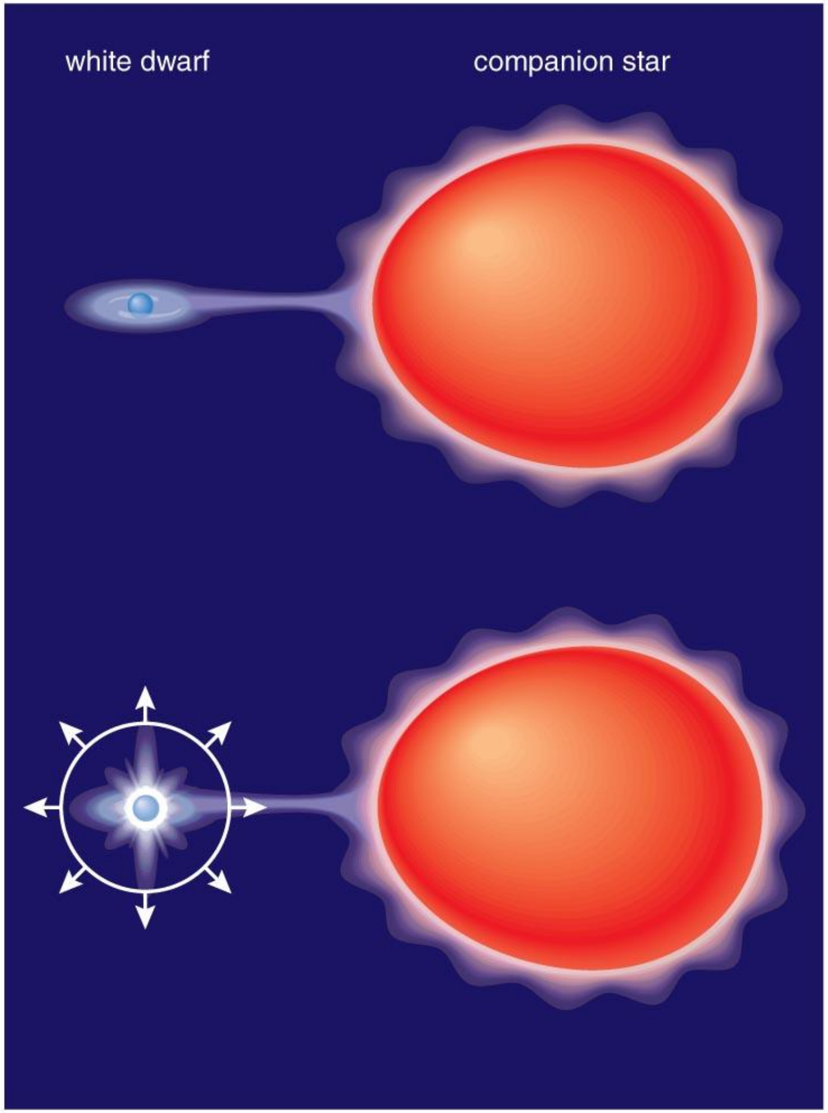
- Mass falling toward a white dwarf from its close binary companion has some angular momentum.
- The matter therefore orbits the white dwarf in an *accretion disk*.

Accretion Disks



- Friction between orbiting rings of matter in the disk transfers angular momentum outward and causes the disk to heat up and glow.

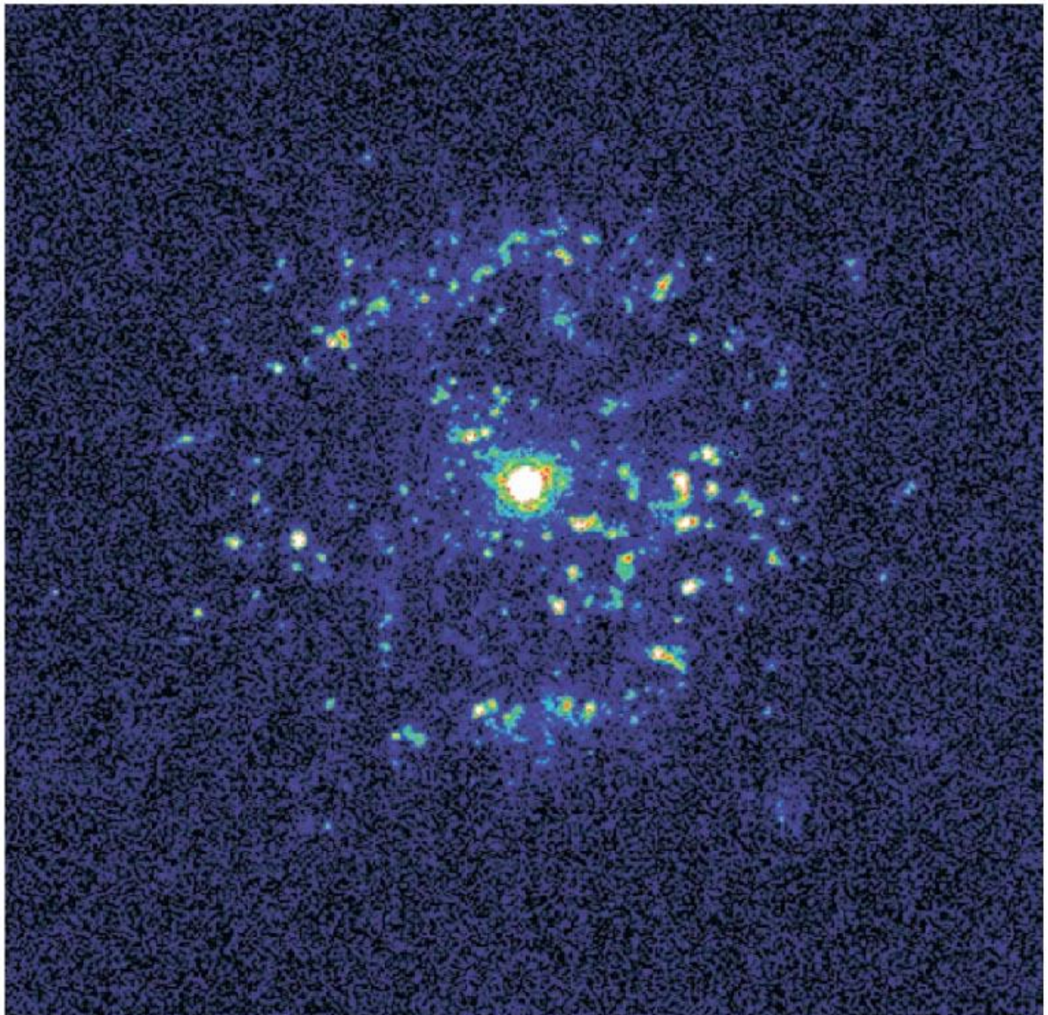
Nova



- The temperature of accreted matter eventually becomes hot enough for hydrogen fusion.

- Fusion begins suddenly and explosively, causing a *nova*.

Nova



- The nova star system temporarily appears much brighter.
- The explosion drives accreted matter out into space.

Thought Question

What happens to a white dwarf when it accretes enough matter to reach the $1.4M_{\text{Sun}}$ limit?

A. It explodes.
B. It collapses into a neutron star.
C. It gradually begins fusing carbon in its core.

Mass versus radius relation

- For objects made of normal matter, radius tends to increase with mass

Mass/radius relation for degenerate star

- Star mass = *M*, radius = *R*
- Gravitational potential energy = $-\frac{3GM^2}{5R}$
- Heisenberg uncertainty: $\Delta x \Delta p \geq \hbar$
- Electron density $n = \frac{3N}{4\pi R^3} \approx \frac{M}{m_p R^3}$

$$\Delta x \approx n^{-1/3} \quad \Delta p \approx \frac{\hbar}{\Delta x} \approx \hbar n^{1/3}$$

- Kinetic energy $\varepsilon = \frac{p^2}{2m_e} \quad K = N\varepsilon = \frac{M}{m_p}\varepsilon \approx \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2}$

Mass/radius relation for degenerate star

- Total energy $$E = K + U \approx \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2} - \frac{GM^2}{R}$$
- Find *R* by minimizing *E*

$$\frac{dE}{dR} \approx -\frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^3} + \frac{GM^2}{R^2} = 0$$

$$R \approx \frac{\hbar^2 M^{-1/3}}{G m_e m_p^{5/3}}$$

- Radius decreases as mass increases